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1971 J. Phys. A: Gen. Phys. 4 535

(http://iopscience.iop.org/0022-3689/4/4/016)

J. Phys. A: Gen. Phys., 1971, Vol. 4. Printed in Great Britain

Harmonic generation due to nonlinear mixing of two strong microwaves in a magnetoplasma†

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MS. received 16th September 1970

Abstract. The effect of intensities of electromagnetic waves on their collisional mixing in an inert gas plasma has been investigated. For strong collisions the generated harmonic and combination frequencies in a helium plasma show a maximum (for an optimum value of wave intensity) while a continuous increase is found for weak collisions. In the presence of a static magnetic field the generation of odd harmonics is not possible with only one mode of the fundamental wave, though even harmonics can be generated if a dc electric field is present. Resonances appear at fundamental and harmonic frequencies.

1. Introduction

The collisional mixing of electromagnetic waves in plasmas has been investigated by several workers in recent years (Ginzburg and Gurevich 1960, Rosen 1961, Epstein 1962, Wetzel and Tang 1965, Sodha and Kaw 1969). The nonlinear phenomena resulting from this mixing are of high practical utility for generation of higher frequency waves (i.e. harmonics and combination frequencies), modulation of waves and plasma diagnostics. The investigations of these workers have been restricted to moderately strong fields which are not high enough to affect the average carrier energy significantly. Consequently at such fields, though the harmonic current densities vary as the nth power of the fundamental field (n being the order of harmonic), the yields are not high.

In a recent paper (Jayaram and Tripathi 1970) the authors investigated the mixing of two microwaves in the presence of a high dc field. The generated difference frequency wave showed a maximum at an optimum value of the dc field. However, the microwaves considered were weak.

Varnum and Desloge (1969) recently reported their investigations on the generation of third harmonics in ionized nitrogen using high-amplitude fundamental waves. They obtained an appreciably high yield of third harmonic current density. However, their analysis did not consider the exact form of the distribution of electron velocities. They approximated it to a Maxwellian one at an elevated temperature.

In this paper we have investigated the interaction of two strong microwaves in a helium plasma when a static magnetic field is applied to it. The analysis is restricted to a slightly-ionized low-temperature plasma because at high temperatures (where the degree of ionization is high) nonlinear effects are less important (Ginzberg and Gurevich 1960). The cross section for collisions of electrons with molecules is taken to be a constant; thus the analysis is valid only for inert gases. The exact form of the isotropic part of the electron distribution function has been obtained by solving the Boltzmann equation in the limit of dominant elastic electron–molecule collisions. Owing to the presence of the static magnetic field the second-order anisotropic part of the distribution is a tensor having finite off-diagonal terms.

† Work partially supported by ESSA (USA).

This general expression for the distribution has been used to evaluate various harmonic and combination frequency components of the current density. The variation of these components with the applied electric and magnetic fields has been presented in the form of graphs for a helium plasma.

2. Electron distribution function

Let us consider a slightly-ionized homogeneous plasma at low temperature. We assume that a static magnetic field is applied along the z axis and two microwaves of frequency ω_1 and ω_2 are propagating in the same direction. Since the gas is slightly ionized we neglect all collisions except the electron–molecule collisions. Expanding the distribution function f in cartesian tensors and retaining terms up to the second-order tensor $\mathbf{f}_2$, the equations governing the f_0 (zero order), $\boldsymbol{f}_1$ (first order) and $\mathbf{f}_2$ (second order) components of f are respectively (Shkarofsky *et al.* 1966)

$$\frac{\partial f_0}{\partial t} - \frac{e}{3mv^2}\frac{\partial}{\partial v}(v^2\boldsymbol{E}.\boldsymbol{f}_1) = \frac{m}{Mv^2}\frac{\partial}{\partial v}\left\{v^2\nu\left(vf_0 + \frac{kT}{m}\frac{\partial f_0}{\partial v}\right)\right\} \tag{1}$$

$$\frac{\partial \boldsymbol{f}_1}{\partial t} - \frac{e\boldsymbol{E}}{m}\frac{\partial f_0}{\partial v} - \frac{e\boldsymbol{H}\times\boldsymbol{f}_1}{mc} - \frac{2e}{5mv^3}\frac{\partial}{\partial v}(v^3\boldsymbol{E}.\mathbf{f}_2) = -\nu\boldsymbol{f}_1 \tag{2}$$

and

$$\left[\frac{\partial \mathbf{f}_2}{\partial t} + \frac{ev}{m}\frac{\partial}{\partial v}\left\{\frac{1}{v}(\boldsymbol{E}\boldsymbol{f}_1 - \tfrac{1}{3}(\boldsymbol{E}.\boldsymbol{f}_1)\mathbf{I})\right\} - \frac{2e\boldsymbol{H}\times\mathbf{f}_2}{mc}\right]_2 = -\nu\mathbf{f}_2 \tag{3}$$

where $\boldsymbol{E}$ and $\boldsymbol{H}$ are the electric and magnetic fields in the system and ν is the electron–molecule collision frequency, the other symbols having their usual meaning. The space gradient terms have been neglected. The collision frequency ν for an inert gas plasma is

$$\nu = \nu_0 u \tag{4}$$

where $u = (m/2kT)^{1/2}v$ and ν_0 is a constant. The electric field $\boldsymbol{E}$, which includes the selfconsistent fields, can be written as

$$\begin{aligned}\boldsymbol{E} = {} & \boldsymbol{E}^1\exp(\mathrm{i}\omega_1 t) + \boldsymbol{E}^2\exp(\mathrm{i}\omega_2 t) + \boldsymbol{E}^{11}\exp(2\mathrm{i}\omega_1 t) + \boldsymbol{E}^{22}\exp(2\mathrm{i}\omega_2 t)\\ & + \boldsymbol{E}^{12}\exp\{\mathrm{i}(\omega_1+\omega_2)t\} + \boldsymbol{E}^{1-2}\exp\{\mathrm{i}(\omega_1-\omega_2)t\}\\ & + \boldsymbol{E}^{112}\exp\{\mathrm{i}(2\omega_1+\omega_2)t\} + \boldsymbol{E}^{11-2}\exp\{\mathrm{i}(2\omega_1-\omega_2)t\} + \dots .\end{aligned} \tag{5}$$

$\boldsymbol{E}_1$ and $\boldsymbol{E}_2$ are the two externally applied fields, the other terms being selfconsistent fields. Similar expressions for the various components of f can be obtained from equation (5) by replacing the E by f and adding a time-independent term to the equation for f_0.

Since in the evaluation of current density we need the $\boldsymbol{f}_1$ part of the distribution function explicity, we solve equation (2) for fundamental components of $\boldsymbol{f}_1$ to obtain

$$f_{1x}{}^{\alpha} = \frac{e}{2m}\frac{\partial f_0{}^0}{\partial v}(A_1{}^{\alpha}B_{\alpha}{}^- + A_2{}^{\alpha}B_{\alpha}{}^+) \tag{6a}$$

and

$$f_{1y}{}^{\alpha} = \frac{e}{2m\mathrm{i}}\frac{\partial f_0{}^0}{\partial v}(A_1{}^{\alpha}B_{\alpha}{}^- - A_2{}^{\alpha}B_{\alpha}{}^+) \tag{6b}$$

where

$$B_{\alpha}{}^{\pm} = \{\nu + \mathrm{i}(\omega_{\alpha} \pm \omega_e)\}^{-1} \qquad \alpha = 1, 2$$

ω_e is the electron cyclotron frequency, and

$$A_1^{\alpha} = E_x^{\alpha} + iE_y^{\alpha} \qquad \text{and} \qquad A_2^{\alpha} = E_x^{\alpha} - iE_y^{\alpha} \tag{7}$$

are the fields of the extraordinary (left-handed circularly polarized) and ordinary (right-handed circularly polarized) modes respectively. It is easily seen from equations (1–3) that the second harmonic and first-order sum and difference frequency components of $\boldsymbol{f}_1$ vanish.

The nature of f_0^0 is governed by equation (1) which can be solved to give

$$f_0^0 = N_0 \exp\left(-\int_0^u \frac{2u\,du}{1+\alpha(u)}\right) \tag{8a}$$

where

$$\alpha(u) = \frac{e^2 M}{12m^2kT}\sum_{\alpha=1}^{2}(A_1^{\alpha}A_1^{\alpha *}B_{\alpha}^{-}B_{\alpha}^{-*} + A_2^{\alpha}A_2^{\alpha *}B_{\alpha}^{+}B_{\alpha}^{+*}) \tag{8b}$$

and

$$N_0 = \left(\frac{m}{2kT}\right)^{3/2} \Big/ \left[4\pi\int_0^{\infty}\left\{u^2\exp\left(-\int_0^u \frac{2u'\,du'}{1+\alpha(u')}\right)\right\}du\right] \tag{8c}$$

is the normalization constant.

In the limit of $\omega_e \to 0$, and $\omega_{1,2}^2 < \nu^2$, equation (8a) reduces to

$$f_0^0 = N_0(u^2+\alpha)^{\alpha}\exp(-u^2) \tag{9}$$

in the presence of a high dc field. Equation (9) is identical with equation (7a) of Jayaram and Tripathi (1970).

In the limit of weak nonlinearity ($\alpha \ll 1$) or weak collisions ($\nu \ll |\omega_1-\omega_e|$, $|\omega_2-\omega_e|$), equation (8a) reduces to the Maxwellian form, while in the opposite limiting case of strong collisions ($\nu \gg |\omega_1+\omega_e|$, $|\omega_2+\omega_e|$) and $\alpha \gg 1$, it assumes the Druyvesteyn form.

In the case of the $2\omega_1+\omega_2$ frequency components of $\boldsymbol{f}_1$ (which will give rise to an electric field $E^{112}\exp\{i(2\omega_1+\omega_2)t\}$), equation (2) can be solved using equation (3) and equation (1) in the limit $2\omega_1 > m\nu/M$ and $(\omega_1+\omega_2) > m\nu/M$ to give

$$\begin{aligned} f_{1x}^{112} + if_{1y}^{112} = {} & \frac{e}{m}(E_x^{112}+iE_y^{112})P_{112}\frac{\partial f_0^0}{\partial v} + \frac{e^3P_{112}}{48m^3}\left[\frac{1}{i\omega_1}\frac{\partial}{\partial v}\left\{\frac{1}{v^2}\right.\right. \\ & \times \frac{\partial}{\partial v}\left(v^2\frac{\partial f_0^0}{\partial v}A_1^1\left[A_2^1A_1^2\left\{B_1^- + B_1^+ + \frac{2\omega_1}{\omega_1+\omega_2}(B_2^- + B_1^+)\right\}\right.\right. \\ & \left.\left.\left.\left. + A_1^1A_2^2\frac{2\omega_1}{\omega_1+\omega_2}(B_2^+ + B_1^-)\right]\right)\right\}\right] + \frac{e^3P_{112}}{40m^3}\frac{1}{v^3}\frac{\partial}{\partial v}\left\{2A_2^1v^4C_{12}\right. \\ & \times \frac{\partial}{\partial v}\left(\frac{1}{v}\frac{\partial f_0^0}{\partial v}A_1^1A_1^2(B_2^- + B_1^-)\right) + \frac{v^4A_1^1}{3H_{12}}\frac{\partial}{\partial v}\left(\frac{1}{v}\frac{\partial f_0^0}{\partial v}\right. \\ & \left. \times \{A_1^1A_2^2(B_2^+ + B_1^-) + A_2^1A_1^2(B_2^- + B_1^+)\}\right) + 2v^4A_2^2A_1^1A_1^1C_{11} \\ & \left. \times \frac{\partial}{\partial v}\left(\frac{1}{v}\frac{\partial f_0^0}{\partial v}B_1^-\right) + \frac{1}{3}\frac{A_1^2v^4}{H_{11}}\frac{\partial}{\partial v}\left(\frac{1}{v}\frac{\partial f_0^0}{\partial v}A_1^1A_2^1(B_1^- + B_1^+)\right)\right\} \end{aligned} \tag{10}$$

where

$$P_{\alpha\beta\gamma} = \{\nu + i(\omega_\alpha+\omega_\beta+\omega_\gamma-\omega_e)\}^{-1}$$

$$C_{\alpha\beta} = \{\nu + i(\omega_\alpha+\omega_\beta-2\omega_e)\}^{-1} \qquad H_{\alpha\beta} = \nu + i(\omega_\alpha+\omega_\beta)$$

and

$$\alpha, \beta, \gamma = 1, 2.$$

The expression for $f_{1x}^{112} - \mathrm{i}f_{1y}^{112}$ is obtained from equation (10) by replacing ω_e by $-\omega_e$ and $A_{1,2}^{1,2}$ by $A_{2,1}^{1,2}$. Other combination frequency components of f_1 can be evaluated in a similar way. We can see from equation (10) that the contribution of the $\mathbf{f}_2$ terms in the combination frequency components of f_1 is of the same order as that of the f_0 terms.

3. Current density

The current density is defined as

$$J = -\frac{4\pi Ne}{3}\int_0^\infty f_1 v^3\, \mathrm{d}v \tag{11}$$

where N is the electron concentration. For the fundamental and $2\omega_1+\omega_2$ frequency components of current density, equation (11) gives the following expressions:

$$J_x^\alpha \pm \mathrm{i}J_y^\alpha = \frac{\omega_p{}^2 A_\alpha{}^\alpha}{12\pi}\left\langle \frac{\partial}{\partial v}(v^3 B_\alpha{}^\mp)\right\rangle \tag{12}$$

and

$$\begin{aligned} J_x^{112} + \mathrm{i}J_y^{112} = {} & \frac{\omega_p{}^2 e^2}{192 m^2} A_1{}^1 \Bigg[A_1{}^2 A_2{}^1 \Bigg\{ \frac{2}{3\mathrm{i}\omega_1}\Bigg\langle M_1(v^3 P_{112})\Bigg(H_1 B_1{}^- B_1{}^+ \\ & + \frac{\omega_1}{\omega_1+\omega_2} D_{12} B_1{}^+ B_1{}^-\Bigg)\Bigg\rangle + \frac{4}{5}\Bigg\langle F_{12} B_2{}^- B_1{}^- M_2\Bigg(v^4 C_{12}\frac{\partial}{\partial v}P_{112}\Bigg)\Bigg\rangle \\ & + \frac{2}{15}\Bigg\langle D_{12} B_2{}^- B_1{}^+ M_2\Bigg(\frac{v^4}{H_{12}}\frac{\partial}{\partial v}P_{112}\Bigg)\Bigg\rangle + \frac{4}{15}\Bigg\langle H_1 B_1{}^+ B_1{}^+ M_2 \\ & \times \Bigg(\frac{v^4}{H_{11}}\frac{\partial}{\partial v}P_{112}\Bigg)\Bigg\rangle\Bigg\} + A_1{}^1 A_2{}^2 \Bigg\{ \frac{2}{3\mathrm{i}(\omega_1+\omega_2)}\langle M_1(v^3 P_{112}) \\ & \times (D_{12} B_1{}^- B_2{}^+)\rangle + \frac{2}{5}\Bigg\langle D_{12} B_2{}^+ B_1{}^- M_2\Bigg(\frac{v^4}{H_{12}}\frac{\partial}{\partial v}P_{112}\Bigg)\Bigg\rangle \\ & + \frac{4}{5}\Bigg\langle B_1{}^- M_2\Bigg(v^4 C_{11}\frac{\partial}{\partial v}P_{112}\Bigg)\Bigg\rangle\Bigg\}\Bigg] \\ & + \frac{\omega_p{}^2}{12\pi}\Bigg\langle \frac{1}{v^2}\frac{\partial}{\partial v}(v^3 P_{112})\Bigg\rangle (E_x^{112} + \mathrm{i}E_y^{112}) \end{aligned} \tag{13}$$

where $\omega_p{}^2 = 4\pi Ne^2/m$ is the electron plasma frequency,

$$M_1 = \frac{1}{v^2}\frac{\partial}{\partial v}v^2\frac{\partial}{\partial v}\frac{1}{v^2}\frac{\partial}{\partial v} \qquad \text{and} \qquad M_2 = \frac{1}{v^2}\frac{\partial}{\partial v}\frac{1}{v}\frac{\partial}{\partial v}$$

are two operators,

$$H_\alpha = \nu + \mathrm{i}\omega_\alpha \qquad D_{\alpha\beta} = 2\nu + \mathrm{i}(\omega_\alpha + \omega_\beta)$$

$$F_{12} = 2\nu + \mathrm{i}(\omega_1 + \omega_2 - 2\omega_e) \qquad \text{and} \qquad \langle\psi\rangle = 4\pi\int_0^\infty v^2\psi f_0{}^0\,\mathrm{d}v$$

denotes the average of a quantity ψ.

The expression for $J_x^{112} - iJ_y^{112}$ is obtained from equation (13) by replacing ω_e by $-\omega_e$ and $A_{1,2}^{1,2}$ by $A_{2,1}^{1,2}$. Again by replacing ω_2 by $-\omega_2$ and $A_{1,2}^2$ by $A_{2,1}^{2*}$ we obtain an expression for the difference frequency component $2\omega_1 - \omega_2$ of the current density. Similarly by replacing ω_2 by ω_1 and $A_{1,2}^2$ by $\frac{1}{3}A_{1,2}^1$ in equation (13) we obtain the following expression for the third harmonic current density:

$$J_x^{111} + iJ_y^{111} = \omega_p^2 \frac{e(A_1^1)^2}{576} A_2^1 \left[\left\langle H_1 B_1^- B_1^+ \left\{ \frac{2}{i\omega_1} (M_1(v^3 P_{111})) + \frac{4}{5} \left(M_2 \left(\frac{v^4}{H_{11}} \frac{\partial P_{11}}{\partial v} \right) \right) \right\} \right\rangle + \frac{12}{5} \left\langle B_1^- M_2 \left(v^4 C_{11} \frac{\partial P_{111}}{\partial v} \right) \right\rangle \right] + \frac{\omega_p^2}{12\pi} \left\langle \frac{1}{v^2} \frac{\partial}{\partial v} (v^3 P_{111}) \right\rangle (E_x^{111} + iE_y^{111}). \tag{14}$$

The first and fifth averages of equation (13) and the first term of the first average in equation (14) are both due to f_0 and all the other terms are due to $\mathbf{f}_2$. At high wave frequencies the contributions due to f_0 and $\mathbf{f}_2$ are of the same order, and in the limit $\omega \gg \nu$ these are in the same phase. At low frequencies the contribution due to f_0 dominates and becomes out of phase by $\pi/2$ with the contribution due to $\mathbf{f}_2$. The resonance effects appearing at $\omega_{1,2} = \omega_e$, $2\omega_1 + \omega_2 = \omega_e$ and $3\omega_1 = \omega_e$ are clearly visible from equations (12–14) though these are suppressed by collisions.

It is to be noted from equations (13 and 14) that, from like modes of two waves, harmonics and sum frequencies cannot be generated, only difference frequency waves are generated. This can be easily understood in terms of the power absorption from the fundamental wave. The power absorption of a circularly polarized wave is time-independent; hence the collision frequency is not modulated. The power absorption due to the mutual interaction of two waves is given by

$$\begin{aligned} \text{Power absorption} &= \mathrm{Re}(\boldsymbol{E}^1) \times \mathrm{Re}(\boldsymbol{J}^2) + \mathrm{Re}(\boldsymbol{E}^2) \times \mathrm{Re}(\boldsymbol{J}^1) \\ &= \tfrac{1}{2}\mathrm{Re}(E^1 J^2 + E^1 J^{2*} + E^2 J^1 + E^{2*} J^1). \end{aligned} \tag{15}$$

For waves having the same circular polarization, that is $E_x^{1,2} = iE_y^{1,2}$ and $J_x^{1,2} = i\,J_y^{1,2}$, the first and third terms of equation (15) vanish. Therefore only difference frequencies are generataed. This result is contradictory to that of Sodha and Kaw (1965) who, neglecting the off-diagonal terms of the $\mathbf{f}_2$ tensor, showed that, even with one mode of a fundamental wave, third harmonics can be generated. As we have neglected the motion of ions the generated frequency must be greater than the ion cyclotron frequency.

We can mention the following special cases.

(i) If $\omega_2 = 0$ (i.e. the second electric field is a dc field) and if we replace $A_{1,2}^2$ by $2A_{1,2}^0$, equation (13) reduces to an expression for the second harmonic current density.

(ii) If $\nu \ll |\omega_{1,2} - \omega_e|$ and α is arbitrary, equation (14) assumes the form

$$J_x^{111} \pm iJ_y^{111} = \frac{\omega_p^2 e^2 A_{1,2}^1 A_{1,2}^1, A_{2,1}^1}{96\pi mkT} (I_1 + I_2). \tag{16}$$

where

$$I_1 = -\frac{16\pi^{-1/2}(1+\alpha)^{-1/2}\nu_0}{(3\omega_1 \mp \omega_e)^2(\omega_1^2 - \omega_e^2)} \left[\frac{1}{5} + \frac{3\pi^{1/2}}{8} i\nu_0(1+\alpha)^{1/2} \times \left\{ \frac{2(\omega_1^2 + \omega_e^2)}{5(\omega_1^2 - \omega_e^2)\omega_1} + \frac{2}{3(\omega_1 \mp \omega_e)} + \frac{1}{30\omega_1} \right\} \right] \tag{17}$$

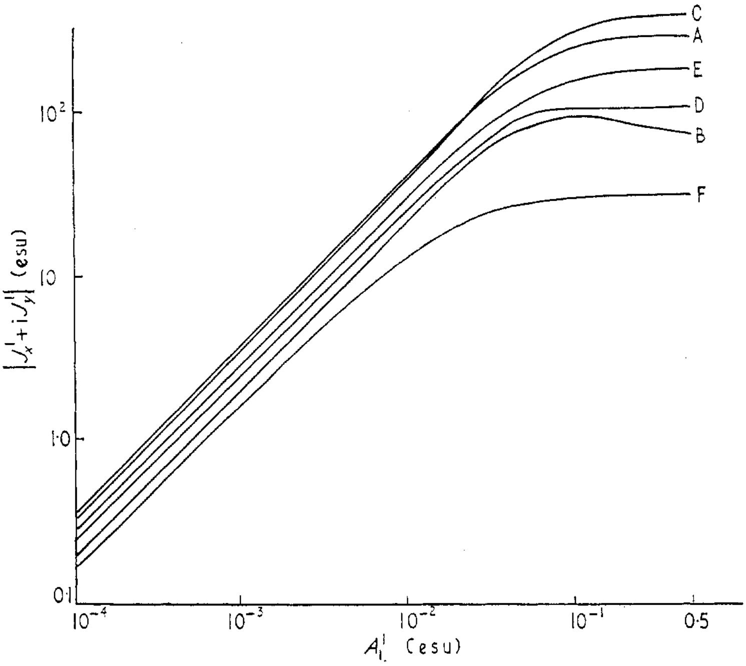

Figure 1. Extraordinary fundamental current density against A_1^1 for $A_2^1 = A_1^2 = A_2^2 = 0{\cdot}05$ esu, $\omega_1 = \omega_e = 10^{10}$ rad s^{-1}. Curves A, C and E correspond to $\nu_0 = 10^9$ s^{-1} and $\omega_2 = 5\times10^9$, $1{\cdot}9\times10^{10}$ and $0{\cdot}99\times10^{10}$ rad s^{-1} respectively and curves B, D and F are for $\nu_0 = 10^8$ s^{-1} and the same ω_2.

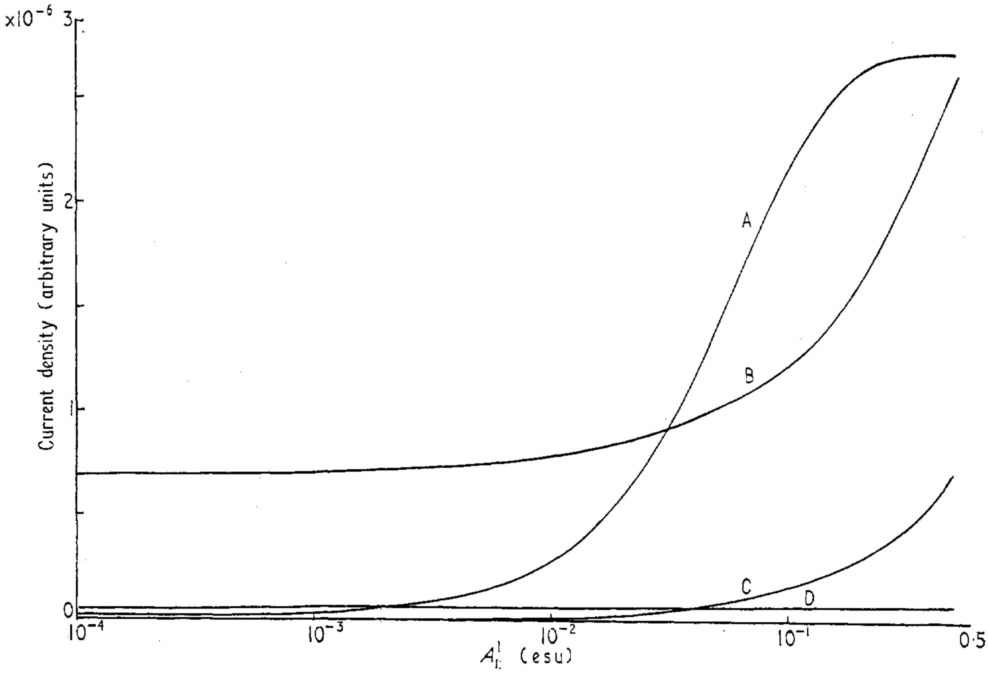

Figure 2. Off-resonance extraordinary current densities against A_1^1 for $\omega_1 = 10^{10}$ rad s^{-1}, $\omega_2 = 5\times10^9$ rad s^{-1}, $A_1^2 = A_2^1 = A_2^2 = 0{\cdot}05$ esu. Curves A and B are for fundamental current densities with $\omega_e = 10^{11}$ rad s^{-1} and $\nu_0 = 10^9$ and 10^8 s^{-1} respectively, curves C and D are for the $2\omega_1 + \omega_2$ frequency current densities with same ω_e and ν_0 and different ordinate.

and

$$I_2 = \frac{16\pi^{-1/2}(1+\alpha)^{-1/2}\nu_0}{5(3\omega_1 \mp \omega_e)^2(\omega_1 \mp \omega_e)^2}\left(\frac{1}{2}+\frac{19}{16}\frac{i\nu_0\pi^{1/2}(1+\alpha)^{1/2}}{(\omega_1 \mp \omega_e)}\right) \tag{18}$$

for $(3\omega_1 - \omega_e) > \nu$, that is, at off-resonance.

At resonance $(3\omega_1 = \omega_e)$ the values of I_1 and I_2 for extraordinary mode are

$$I_1 = \frac{8\pi^{-1/2}(1+\alpha)^{-3/2}}{\nu_0(\omega_1^2-\omega_e^2)}\left\{\frac{1}{5}+\frac{i\nu_0(1+\alpha)^{1/2}\pi^{1/2}}{4\omega_1}\left(\frac{1}{30}+\frac{2}{5}\frac{\omega_1^2+\omega_e^2}{\omega_1^2-\omega_e^2}\right)\right\} \tag{19}$$

and

$$I_2 = \frac{4\pi^{-1/2}(1+\alpha)^{-3/2}}{(\omega_1 \mp \omega_e)^2\nu_0}\left(\frac{1}{2}+\frac{3}{8}\pi^{1/2}\frac{i\nu_0(1+\alpha)^{1/2}}{(\omega_1 \mp \omega_e)}\right). \tag{20}$$

Equations (16–18) show that the third harmonic current density at off-resonance varies as $(1+\alpha)^{-1/2}\, A_{1,2}{}^1\, A_{1,2}{}^1 A_{2,1}{}^1$ with the amplitude of the fundamental wave. This shows that the variation at high fields will be as the second power of the amplitude of the fundamental wave, while at weak fields it will vary as the cube of the amplitude. At resonance the variation is of the form $(1+\alpha)^{-3/2}\, A_{1,2}{}^1 A_{1,2}{}^1 A_{2,1}{}^1$, which becomes independent of the fundamental wave amplitude of very high fields.

4. Discussion and conclusions

In order to study the effect of wave amplitude and static magnetic field on high frequency generation, the averages of equations (12) and (13) have been evaluated numerically on an ICT1909 computer for a helium plasma. The general case of combination frequency generation has been considered for two values of ν_0 (i.e. $\nu_0 = 10^8$ and $10^9\ \mathrm{s}^{-1}$ where $\nu_0 = qN_m(2kT/m)^{1/2}$, q being the constant collision cross section and N_m the molecular density) and the results are shown in the form of graphs.

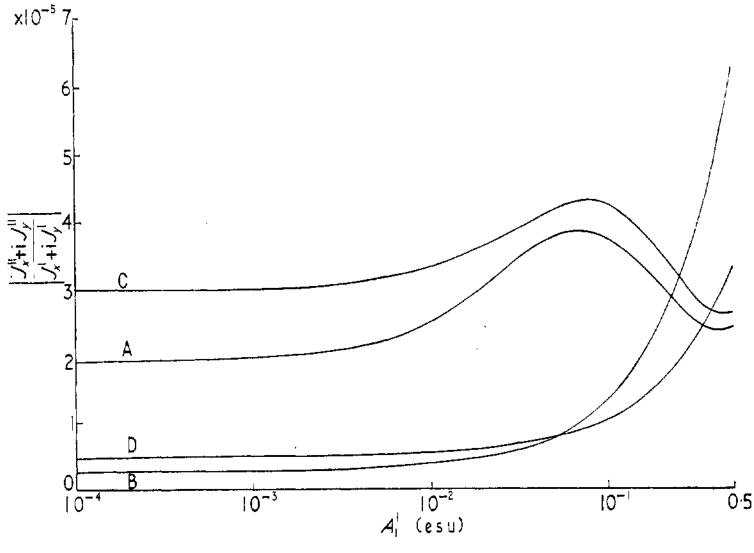

Figure 3. Ratio of extraordinary second harmonic current density to the fundamental current density against $A_1{}^1$ for $A_1{}^2 = A_2{}^1 = A_2{}^2 = 0{\cdot}05$ esu, $\omega_1 = 10^{10}\ \mathrm{rad\ s^{-1}}$. Curves A and B are for $\nu_0 = 10^9$ and $10^8\ \mathrm{s^{-1}}$ respectively at $\omega_e = 10^{10}\ \mathrm{rad\ s^{-1}}$ and curves C and D are for the same parameters at $\omega_e =$

In figure (1) we see that the fundamental current densities (equation 12) increase with the amplitudes of the fundamental waves as long as the effect of wave amplitude on electron temperature is small. At high field strengths of fundamental waves, when resonance conditions are not satisfied and collisions are weak (figure 2), the current density goes on increasing with the field; otherwise the current density gets saturated. This saturation is due to the inverse dependence of the conductivity at cyclotron resonance on the electron collision frequency. Owing to the increase in collision frequency with field strength, the conductivity decreases with increasing field strength.

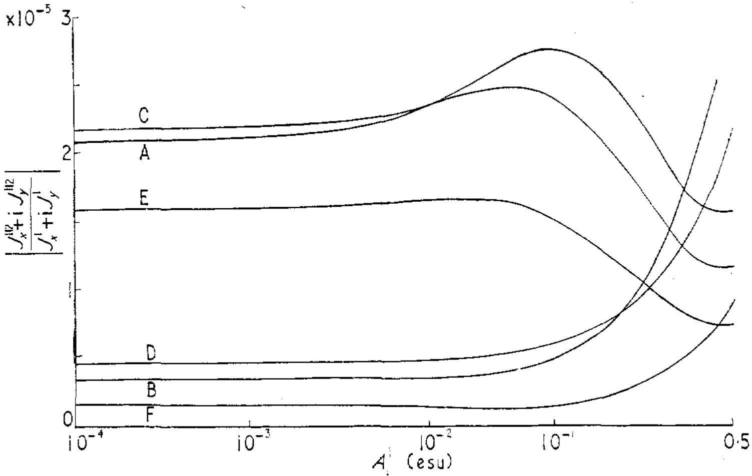

Figure 4. Normalized $2\omega_1+\omega_2$ frequency extraordinary current density against $A_1{}^1$ for $A_1{}^2 = A_2{}^1 = A_2{}^2 = 0{\cdot}05$ esu, $\omega_1 = \omega_e = 10^{10}$ rad s^{-1}. Curves A, B and E are for $\omega_2 = 0{\cdot}5\times10^{10}$, $1{\cdot}9\times10^{10}$ and $0{\cdot}99\times10^{10}$ rad s^{-1} respectively at $\nu_0 = 10^9$ s^{-1} and curves B, D and F are for the same parameters at $\nu_0 = 10^8$ rad s^{-1}.

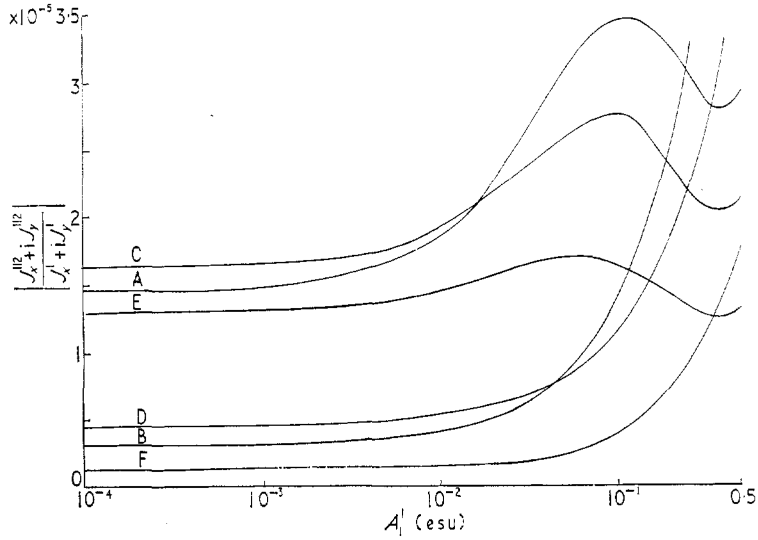

Figure 5. Normalized $2\omega_1+\omega_2$ frequency extraordinary current density against $A_1{}^1$ for $A_1{}^2 = A_2{}^1 = A_2{}^2 = 0{\cdot}1$ esu, $\omega_1 = \omega_e = 10^{10}$ rad s^{-1}. Curves A, C and E are for $\omega_2 = 0{\cdot}5\times10^{10}$, $1{\cdot}9\times10^{10}$ and $0{\cdot}99\times10^{10}$ rad s^{-1} respectively and $\nu_0 = 10^9$ s^{-1}; curves B, D and F are for the same parameters at $\nu_0 = 10^8$ s^{-1}.

The ratios of nonlinear extraordinary second harmonic and second-order combination frequency current densities to the fundamental current density (figures 3–5) show a maximum followed by a minimum for the high collision frequency case (curves A and C of figure 3 and curves A, C and E of figures 4 and 5), while these show a rather continuous rise for the low collision frequency one (curves B and D of figure 3 and curves B, D and F of figures 4 and 5). For strong collisions the fall beyond the maximum is due to the decrease in harmonic current density with collision frequency, on condition that the latter exceeds the electron cyclotron and wave frequencies. Since at high fields the rise in collision frequency with field strength is rapid, this condition is satisfied, hence collisions dominate the harmonic current variation with field strength resulting in this fall in the current density. At high fields the isotropic part of the distribution function assumes the Druyvesteyn form and collisions vary slowly with field strength thereby increasing the ratio again. For weak collisions the harmonic current increases with collision frequency; hence it also increases more rapidly with the electric field.

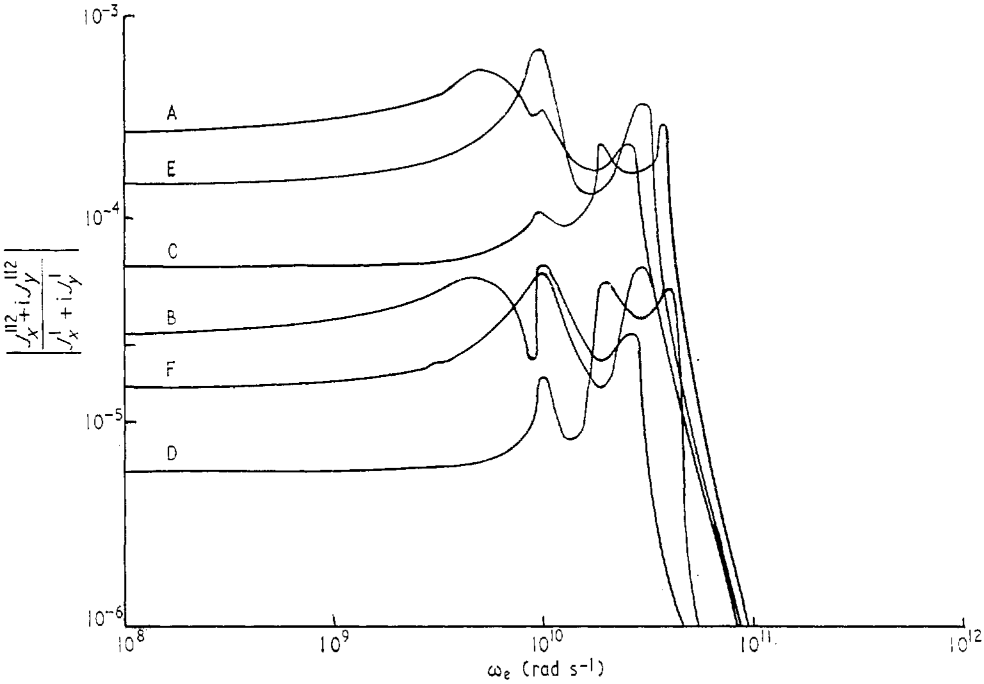

Figure 6. Normalized extraordinary $2\omega_1+\omega_2$ frequency current density against ω_e for $A_1^1 = 10^{-3}$ esu, $A_1^2 = A_2^1 = A_2^2 = 0{\cdot}01$ esu, $\omega_1 = 10^{10}$ rad s^{-1}. Curves A, C and D are for $\nu_0 = 10^9$ s^{-1} and $\omega_2 = 0{\cdot}5\times10^{10}$, $1{\cdot}9\times10^{10}$ and $0{\cdot}99\times10^{10}$ rad s^{-1} respectively; curves B, D and F are for $\nu_0 = 10^8$ s^{-1} and the same ω_2.

The variation of the nonlinear harmonic current density ratio (figures 6 and 7) with static magnetic field shows maxima and minima in the resonance region. For very weak magnetic fields this ratio is unaffected while for high magnetic fields it decreases sharply. This is true for both weak and strong collisions as in these regions the resonance effects are suppressed by collisions.

In conclusion we can say that the collisional mixing of strong electromagnetic waves in a weakly ionized homogeneous plasma leads to the generation of harmonics; only odd harmonics are generated in the absence of a dc field and in the presence of a dc field even harmonics are also possible. Generation of odd harmonics is not

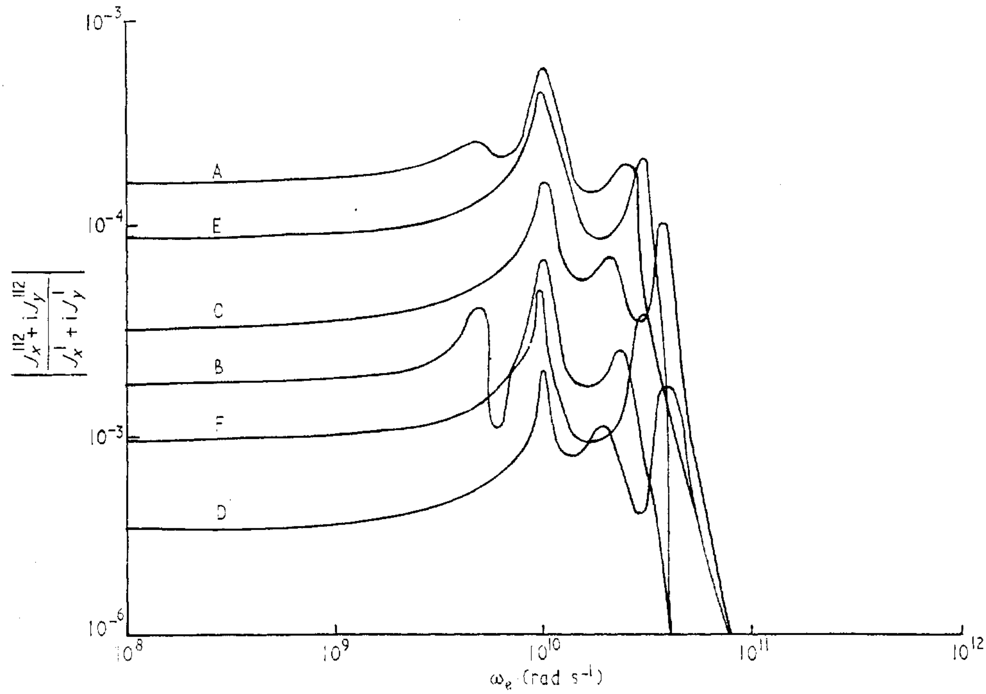

Figure 7. Normalized extraordinary $2\omega_1+\omega_2$ frequency current density against ω_e for $A_1^2 = 10^{-3}$ esu, $A_2^1 = A_1^1 = A_2^2 = 0{\cdot}01$ esu, $\omega_1 = 10^{10}$ rad s^{-1}. Curves A, C and D are for $\nu_0 = 10^9$ s^{-1} and $\omega_2 = 0{\cdot}5\times10^{10}$, $1{\cdot}9\times10^{10}$ and $0{\cdot}99\times10^{10}$ rad s^{-1} respectively, curves B, D and F are for $\nu_0 = 10^8$ s^{-1} and the same ω_2.

possible with only one mode of the fundamental wave though even harmonics can be generated if a dc field is present. The desired mode of even harmonics can be obtained from the mode of same polarization of the fundamental wave. The effect of high wave intensities on harmonic generation comes out to be high and is governed by the effectiveness of the collisions.

The analysis is applicable to velocity-dependent scattering cross sections also and numerical computations can be performed. This velocity dependence of the scattering cross section is revealed in the dependence of harmonic power on incident power. Conversely the variation of harmonic power with incident power can be used as a diagnostic tool.

The generation of harmonics in a dense plasma, where the ordinary mode of fundamental waves does not propagate, is possible due to the interaction of helicons, if a dc field is present.

We have not evaluated the selfconsistent electric field for which the wave equation has to be solved. Papa and Haskell (1966) have solved the wave equation for fundamental waves. Using their solution for the fundamental waves and applying proper boundary conditions one can find the electric intensity in a slab geometry. This will be reported shortly.

Acknowledgments

We are grateful to Professor M. S. Sodha and Dr B. Buti for their valuable guidance during the course of the investigation. The financial support of CSIR (India) and ESSA (USA) is thankfully acknowledged by S.J. and V.K.T. respectively.

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